RECENT APPLICATIONS OF KOTANI THEORY

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In this talk we summarize some of the results that are often referred to as "Kotani theory" and discuss some recent applications of this theory.

We consider discrete one-dimensional ergodic Schrödinger operators

$$[H_\omega \psi](n) = \psi(n+1) + \psi(n-1) + V_\omega(n)\psi(n)$$

in $\ell^2(\mathbb{Z})$, where (Ω, T, μ) is an invertible ergodic dynamical system, $f : \Omega \to \mathbb{R}$ is bounded and measurable, and $V_\omega(n) = f(T^n\omega)$. The spectral properties of H_ω are μ-almost surely independent of the parameter ω. In particular, there exists a set Σ_{ac} such that the absolutely continuous spectrum of H_ω is equal to Σ_{ac} for μ-almost every ω.

The main output of Kotani theory is a description of the set Σ_{ac} in terms of the Lyapunov exponent. Let $E \in \mathbb{C}$ and

$$A^E(\omega) = \begin{pmatrix} E - f(\omega) & -1 \\ 1 & 0 \end{pmatrix}.$$

Define the Lyapunov exponent by

$$\gamma(E) = \lim_{n\to\infty} \frac{1}{n} \int \log \|A_n^E(\omega)\| \, d\mu(\omega)$$

where

$$A_n^E(\omega) = A^E(T^{n-1}\omega) \cdots A^E(\omega).$$

Then, it follows from results of Ishii, Pastur, and Kotani that Σ_{ac} is equal to the essential closure of the set of E's for which the Lyapunov exponent vanishes. Kotani has also found sufficient conditions for the absence of absolutely continuous spectrum and the presence of (purely) absolutely continuous spectrum.

The applications we present include the absence of absolutely continuous spectrum for general underlying dynamics and rough f as well as the presence of purely absolutely continuous spectrum for the subcritical almost Mathieu operator.

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