LYAPUNOV EXPONENTS OF DETERMINISTIC PRODUCTS OF MATRICES ARISING IN THE STUDY OF SCHRDINGER OPERATORS

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In this talk we consider discrete one-dimensional ergodic Schrödinger operators

$$[H_\omega \psi](n) = \psi(n+1) + \psi(n-1) + V_\omega(n)\psi(n)$$

in $\ell^2(\mathbb{Z})$, where the potentials V_ω are generated as follows. We consider the ergodic dynamical system (Ω, T, μ), where $\Omega = \{0,1\}^{\mathbb{Z}}$, $T : \Omega \to \Omega$, $[T\omega](n) = \omega(n+1)$, and $\mu = \mathbb{P}^{\mathbb{Z}}$ with $\mathbb{P}(\{0\}) = p \in (0,1)$. Consider the metric d on Ω given by

$$d(\omega, \omega') = \sum_{n \in \mathbb{Z}} e^{-|n|} |\omega(n) - \omega'(n)|.$$

Let $f : \Omega \to \mathbb{R}$ be Hölder continuous and $\lambda > 0$. Define for $\omega \in \Omega$ and $n \in \mathbb{Z}$,

$$V_\omega(n) = \lambda f(T^n \omega).$$

Let $E \in \mathbb{C}$ and

$$A^E(\omega) = \begin{pmatrix} E - \lambda f(\omega) & -1 \\ 1 & 0 \end{pmatrix}$$

Define the Lyapunov exponent by

$$\gamma(E) = \lim_{n \to \infty} \frac{1}{n} \int \log \| A_n^E(\omega) \| \, d\mu(\omega)$$

where

$$A_n^E(\omega) = A^E(T^{n-1}\omega) \cdots A^E(\omega)$$

Our main result, which is joint with Artur Avila (Paris), says that for λ small enough, the Lyapunov exponent is strictly positive for all but finitely many values of E. Our proof is based on a recent generalization of the classical Fürstenberg Theorem to the correlated situation, which is due to Bonatti, Gómez-Mont, and Viana, together with some results from the inverse spectral theory of periodic operators.

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