# LOCALIZATION FOR SCHRÖDINGER OPERATORS WITH POISSON RANDOM POTENTIAL

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Consider an electron moving in an amorphous medium with randomly placed identical impurities, each impurity creating a local potential. For a fixed configuration of the impurities, described by the countable set $X \subset \mathbb{R}^d$ giving their locations, this motion is described by the Schrödinger equation $-i\partial_t \psi_t = H_X \psi_t$ with the Hamiltonian

$$H_X := -\Delta + V_X \quad \text{on} \quad \mathrm{L}^2(\mathbb{R}^d), \tag{1}$$

where the potential is given by

$$V_X(x) := \sum_{\zeta \in X} u(x - \zeta), \tag{2}$$

with $u(x-\zeta)$ being the single-site potential created by the impurity placed at $\zeta$. Since the impurities are randomly distributed, the configuration $X$ is a random countable subset of $\mathbb{R}^d$, and hence it is modeled by a point process on $\mathbb{R}^d$.

The *Poisson Hamiltonian* is the random Schrödinger operator $H_{\mathbf{X}}$ in (1) with $\mathbf{X}$ a Poisson process on $\mathbb{R}^d$ with density $\varrho > 0$. The potential $V_{\mathbf{X}}$ is then a *Poisson random potential*. Poisson Hamiltonians may be the most natural random Schrödinger operators in the continuum as the distribution of impurities in various samples of material is naturally modeled by a Poisson process. A mathematical proof of the existence of localization in two or more dimensions has been a long-standing open problem.

In this lecture I discuss the following theorem proved by F. Germinet, P. Hislop and myself:

**Theorem 1** (Germinet, Hislop and Klein)**.** *Let $H_{\mathbf{X}}$ be a Poisson Hamiltonian on $\mathrm{L}^2(\mathbb{R}^d)$ with density $\varrho > 0$. Then there exist $E_0 = E_0(\varrho) > 0$ and $m = m(\rho) > 0$ for which the following holds $\mathbb{P}$-a.e.: The operator $H_{\mathbf{X}}$ has pure point spectrum in $[0, E_0]$ with exponentially localized eigenfunctions with rate of decay $m$, i.e., if $\phi$ is an eigenfunction of $H_{\mathbf{X}}$ with eigenvalue $E \in [0, E_0]$ we have*

$$\|\chi_x \phi\| \le C_{\mathbf{X},\phi}\, e^{-m|x|}, \quad \textit{for all } x \in \mathbb{R}^d. \tag{3}$$

*Moreover, there exist $\tau > 1$ and $s \in ]0,1[$ such that for all eigenfunctions $\psi, \phi$ (possibly equal) with the same eigenvalue $E \in [0, E_0]$ we have*

$$\|\chi_x \psi\|\, \|\chi_y \phi\| \le C_{\mathbf{X}} \|T^{-1}\psi\| \|T^{-1}\phi\|\, e^{\langle y \rangle^\tau} e^{-|x-y|^s}, \quad \textit{for all } x, y \in \mathbb{Z}^d. \tag{4}$$

*In particular, the eigenvalues of $H_{\mathbf{X}}$ in $[0, E_0]$ have finite multiplicity, and $H_{\mathbf{X}}$ exhibits dynamical localization in $[0, E_0]$, that is, for any $p > 0$ we have*

$$\sup_t \|\langle x \rangle^p e^{-itH_{\mathbf{X}}} \chi_{[0,E_0]}(H_{\mathbf{X}}) \chi_0\|_2^2 < \infty. \tag{5}$$