LOCALIZATION FOR RANDOM SCHRÖDINGER OPERATORS WITH SINGULAR CONTINUOUS PROBABILITY DISTRIBUTIONS

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We consider random Schrödinger operators with single site potential u and random amplitudes $\boldsymbol{\omega} = \{\boldsymbol{\omega}_\zeta\}_{\zeta \in \mathbb{Z}^d}$, where u is a nonnegative, nonzero L^∞-function on $\mathbb{R}^d$ with compact support, and $\boldsymbol{\omega} = \{\boldsymbol{\omega}_\zeta\}_{\zeta \in \mathbb{Z}^d}$ is a family of independent, identically distributed random variables, such that the common probability distribution μ satisfies $0 \in \operatorname{supp} \mu \subset [0,1]$ and $\bar{\mu} := \mathbb{E}\{\boldsymbol{\omega}_0\} > 0$.

The μ-Anderson Hamiltonian is

$$H_{\boldsymbol{\omega}} := -\Delta + V_{\boldsymbol{\omega}} \quad \text{on} \quad \mathrm{L}^2(\mathbb{R}^d), \quad V_{\boldsymbol{\omega}}(x) := \sum_{\zeta \in \mathbb{Z}^d} \boldsymbol{\omega}_\zeta \, u(x-\zeta). \tag{1}$$

The μ-Poisson Hamiltonian is

$$H_{\boldsymbol{\omega},\mathbf{X}} := -\Delta + V_{\boldsymbol{\omega},\mathbf{X}} \quad \text{on} \quad \mathrm{L}^2(\mathbb{R}^d), \quad V_{\boldsymbol{\omega},\mathbf{X}}(x) := \sum_{\zeta \in \mathbf{X}} \boldsymbol{\omega}_\zeta \, u(x-\zeta), \tag{2}$$

with $\mathbf{X}$ a Poisson process on $\mathbb{R}^d$ with density $\varrho > 0$.

A measure μ will be called q-log-Hölder continuous ($q > 0$) if for some constants C and $s_0 \in (0,1)$ we have

$$Q_\mu(s) := \max_{x \in \mathbb{R}} \mu\{x + [0,s]\} \le \frac{C}{(|\log s|)^q} \quad \text{for all} \quad s \in (0, s_0]. \tag{3}$$

In this lecture I discuss the following theorem proved by F. Germinet and myself:

Theorem 1 (Germinet and Klein)**.** *Let $H_{\boldsymbol{\omega}}$ be either the μ-Anderson Hamiltonian or the the μ-Poisson Hamiltonian. Suppose*

$$\mu = \alpha \mu_q + \beta \nu$$

where

- *μ_q is a qd-log-Hölder continuous probability measure with $q > \frac{1}{3}$.*
- *ν is an arbitrary probability measure .*
- *$0 < \alpha \le 1$, $\alpha + \beta = 1$.*

Then $H_{\boldsymbol{\omega}}$ exhibits localization at the bottom of the spectrum, that is, there exist $E_0 > 0$ and $m > 0$ such that :

- *The following holds with probability one: $H_{\boldsymbol{\omega}}$ has pure point spectrum in $[0, E_0]$ with exponentially decaying eigenfunctions and eigenvalues of finite multiplicity.*
- *There is strong dynamical localization in the form: there is $s > 0$ such that*

$$\mathbb{E}\left\{ \sup_{t \in \mathbb{R}} \left\| \langle x \rangle^{\frac{p}{2}} e^{-itH_{\boldsymbol{\omega}}} \chi_{[0,E_0]}(H_{\boldsymbol{\omega}}) \chi_0 \right\|_2^{\frac{2s}{p}} \right\} < \infty \quad \textit{for all } \; p \ge 1. \tag{4}$$