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Averages of spectral measures for random operators

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We consider a probability space $(\Omega, \mathcal{C}, \mathbb{P})$ and a self-adjoint operator valued random variable A, taking values in the space of linear operators on a separable Hilbert space $\mathcal{H}$, on it. Suppose q^ω is a real valued random variable which is independent of A. We consider a unit vector $\phi \in \mathcal{H}$ and look at the family of operators $H^\omega = A^\omega + q^\omega P_\phi$, where P_ϕ is the orthogonal projection onto the ray generated by ϕ and q^ω is distributed according to a probability measure μ (on $\mathbb{R}$).

We use the following theorem on the characterization of the continuity properties of a probability measure on $\mathbb{R}$, for which we define it to be uniformly Hölder continuous with exponent $0 < \alpha \leq 1$, whenver

$$\sup_{x\in\mathbb{R}} \sup_{a>0} \frac{\mu((x-a, x+a))}{a^\alpha} < \infty.$$

Theorem 0.1. *Suppose μ is a probability measure on $\mathbb{R}$ and ψ is a function on $\mathbb{R}$ which is non-negative, even and satisfies $|\psi| + |x\psi'| \in L^1(\mathbb{R})$. Then μ is uniformly Holder continuous iff*

$$\sup_{x\in\mathbb{R}} \sup_{a>0} \frac{1}{a^\alpha} |(\psi_a * \mu)(x)| < \infty, .$$

where $\psi_a(x) = \psi(x/a)$.

Using the above theorem, we prove that for the operators H_ω defined above:

Theorem 0.2. *Consider H^ω as above. Let E_{H^ω} denote the spectral family associated with the self-adjoint operator and define the measures*

$$\nu_\omega = \int \langle \phi, E_{H^\omega}(\dot{)}\phi \rangle \; d\mu(q^\omega).$$

Then ν^ω is uniformly Hölder continuous whenever μ is uniformly Hölder continuous.