# Asymptotics of Orthogonal Polynomials, Quasiperiodic Jacobi Matrices, and Random Matrices

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We discuss recent results on asymptotics of orthogonal polynomials, stressing their spectral aspects and similarity in two cases considered. They are polynomials orthonormal on a finite union of disjoint intervals with respect to the Szegö weight and polynomials orthonormal on $\mathbb{R}$ with respect to varying weights and having the same union of intervals as the set of oscillations of asymptotics. In both cases we construct double infinite finite band Jacobi matrices with generically quasiperiodic coefficients and show that each of them is an isospectral deformation of another. We find also the Integrated Density of States and the Lyapunov Exponent of Jacobi matrices via the quantities, entering the asymptotics.

Basing on these results and their certain developments, we study then the variance and the characteristic functional of linear eigenvalue statistics of unitary invariant Matrix Models of $n \times n$ Hermitian matrices as $n \to \infty$. Assuming that the test function of statistics is smooth enough, we show first that if the support of the Density of States of the model consists of $q \geq 2$ intervals, then in the global regime the variance of statistics is a quasiperiodic function of $n$ as $n \to \infty$ generically in the potential, determining the model. We show next that the exponent of the characteristic functional in general is not $1/2 \times$ variance, as it should be if the Central Limit Theorem would be valid, and we find the asymptotic form of the characteristic functional in certain cases. We give also the asymptotic form of the variance of linear eigenvalue statistics of hermitian matrix models in the intermediate and the global asymptotic regimes of Random Matrix Theory and argue that the

Central Limit Theorem is valid in the intermediate regime and is not valid in the local regime.