ON THE MULTIPLE POINTS OF RANDOM WALKS

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Let $\{S_n\}_{n=0}^{\infty}$ be a random walk on $\mathbb{Z}^d$ starting at the rogin. The p-multiple point range at time n of the random walk, denoted by $Q_n^{(p)}$, means the number of distinct points entered exactly p times by the random walk in the first n steps.

If the random walk is transient, Pitt proved that $Q_n^{(p)}$ obeys the law of large numbers. Moreover, for the two dimensional simple random walk, Flatto proved the law of large numbers of $Q_n^{(p)}$.

The central limit theorem for the multiple point range of two or more dimensional adapted random walks was almost established by Hamana. According to his results, the central limit theorem for $Q_n^{(p)}$ holds in the three or more dimensional cases. However it dose not hold in the two dimensional case since the limit distribution is not Gaussian.

We next comsider the large deviation principles for the multiple point range of random walks. Similarly to the range of random walks, the large devition in the upwards direction can be obtained.

Theorem. *If the random walks is simple and two or more dimensional, we have that for $p \geqq 1$ and $x \neq 1/p$*

$$\psi_p(x) = -\lim_{n\to\infty} \frac{1}{n} \log P[Q_n^{(p)} \geqq xn]$$

exists. The function ψ_p has the following property:

(1) $\psi_p(x) = 0$ *if* $x \leqq \gamma^2(1-\gamma)^{p-1}$

(2) $0 \leqq \psi_p(x) < \infty$ *if* $\gamma^2(1-\gamma)^{p-1} < x < 1/p$

(3) $\psi_p(x) = \infty$ *if* $x > 1/p$

(4) ψ_p *is continuous and convex on* $(-\infty, 1/p]$

References

[1] L. Flatto, *The multiple range of two-dimensional recurrent walk*, Ann. Probab. **4** (1976), 229–248.

[2] Y. Hamana, *On the central limit theorem for the multiple point range of random walk*, J. Fac. Sci. Univ. Tokyo Sect. IA Math. **39** (1992), 339–363.

[3] ———, *On the multiple point range of three dimensional random walks*, Kobe J. Math. **12** (1995), 95-122.

[4] ———, *The fluctuation result for the multiple point range of two dimensional recurrent random walks*, Ann. Probab. **25** (1997), 568–639.

[5] J.H. Pitt, *Multiple points of transient random walk*, Proc. Amer. Math. Soc. **43** (1974), 195–199.