Wegner estimates for Anderson type random surface potentials

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We consider a Schrödinger operator on the Hilbert space $L^2(\mathbb{R}^d)$ $(d \geq 2)$ with the random potential concentrated near a surface $\mathbb{R}^{d_1} \times \{0\} \subset \mathbb{R}^d(1 \leq d_1 < d)$; $H^\omega := -\sum_{\nu=1}^{d} \partial^2/\partial x_\nu^2 + \sum_{k\in\mathbb{Z}^{d_1}} \omega_k u(x_1 - k, x_2)$ for $x = (x_1, x_2) \in \mathbb{R}^{d_1} \times \mathbb{R}^{d-d_1}$. The single-site potential $u(x)$ is a non-zero and non-negative function less than or equal to 1 with a support included in a unit cube $\Lambda_1(0) := (-1/2, 1/2)^d$. The coupling constants $\{\omega_k\}_{k\in\mathbb{Z}^{d_1}}$ are independently identically distributed random variables such that the support of its common probability distribution μ is the finite interval $[\omega_-, 0]$ for some $\omega_- \in (-\infty, 0)$. We improved Wegner estimate given in [2,4] as follows: let χ_{S_L} be the characteristic function of a strip $S_L := (-L/2, L/2)^{d_1} \times \mathbb{R}^{d-d_1}$C $H_L^\omega := -\Delta + V^\omega \chi_{S_L}$, and $n(I; H_L^\omega)$ be the number of eigenvalues of H_L^ω in an interval $I \subset (-\infty, 0)$.

Theorem 1. *Assume that μ is α-Hölder continuous for $\alpha \in (0, 1]$. Then, for any $E < 0$ there exists $C \in (0,\infty)$ such that*

$$\mathbb{E}\{n([E - \eta, E + \eta]; H_L^\omega)\} \leq C\eta^a L^{d_1}$$

for any $0 < \eta < |E|/2$ and $L \in 2\mathbb{N} + 1$.

The method described in [1] and the recent spectral averaging technique [3] are used for the proof. We can also prove the α-Hölder continuity of the integrated density of surface states $N_s(E)$ for $E < 0$ (see [4]), where

$$N_s(E) := \lim_{L\to\infty} \frac{1}{L^{d_1}} n((-\infty, E]; H_{D,S_L}^\omega)$$

and H_{D,S_L}^ω is the Dirichlet restriction of $H^\omega$to S_L.

Corollary 1. *For any $E < 0$, there exists $C \in (0,\infty)$ such that*

$$N_s(E + \eta) - N_s(E - \eta) \leq C\eta^\alpha.$$

for any $0 < \eta < |E|/2$.

Moreover, by applying the multiscale analysis according to [5], one can show Anderson localization for negative energies near the bottom of the spectrum.

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[5] P.Stollmann, *Caught by disorder, Bound states in random media*, Birkhäuser, Boston, 2001.