The energy level statistics for the Anderson tight binding model
- Statement of a conjecture -

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Let us consider the Anderson tight binding model $H_\omega = -\Delta + V_\omega$, where $(\Delta u)(x) = \sum_{|y-x|=1} u(y)$, is the discretized Laplacian, and $V_\omega = \{V_\omega(x)\} \in \mathbb{Z}^d$ is the random potential consisting of i.i.d. random variables. For $L = 1, 2, \ldots$, let $\Lambda := \Lambda_L = [0, L]^d \cap \mathbb{Z}^d$, and consider $H^\Lambda_\omega := \chi_\Lambda H_\omega \chi_\Lambda$, the restriction of $H_\omega$ to the hypercube $\Lambda$ with the Dirichlet boundary condition. Now let $E_1^\Lambda(\omega) \leq \cdots \leq E_n^\Lambda(\omega)$, $n = |\Lambda|$, be the eigenvalues of $H^\Lambda_\omega$. At this point, we assume that each random variable $V_\omega(x)$ has absolutely continuous distribution with a bounded density $\rho(v)$. We also assume that its upperbound $\|\rho\|_\infty$ is small. Smallness of $\|\rho\|_\infty$ is assumed because it implies the exponential decay of the fractional moment of the Green’s function $G^\Lambda(z; x, y) = (H^\Lambda_\omega - z)^{-1}(x, y)$. (See [1], [2] for the exact formulation.)

We now consider the rescaled spectrum of $H^\Lambda_\omega$ expressed as a point process on $\mathbb{R}$: $\xi^\omega(\Lambda; E)(dx) := \sum_j \delta_{\xi^\omega_j(\Lambda; E)}(dx)$, with $\xi^\omega_j(\Lambda; E) = |\Lambda|(E_j^\Lambda(\omega) - E)$. It was proved in [3] that if the integrated density of states $N(E)$ of $H_\omega$ is differentiable at $E$ with $n(E) = dN/dE$, then as $L \to \infty$, the probability law of $\xi^\omega(\Lambda; E)$ converges weakly to that of the stationary Poisson point process with mean density $n(E)$. Then a question arises: Can we compare the individual spectrum of $H^\Lambda_\omega$ with the typical realization of a nice point process on $\mathbb{R}$?

For this purpose, we need to "unfold" the spectrum. Let us assume that $N(E)$ is of $C^1$ and $n(E) > 0$ everywhere on $(\inf \Sigma, \sup \Sigma)$, where $\Sigma \subset \mathbb{R}$ is the closed set such that $\text{supp}(H_\omega) = \Sigma$ a.s.. Now let us call $e^\Lambda_\omega(\omega) := |\Lambda| \cdot N(E_j^\Lambda(\omega)) \in (0, |\Lambda|)$ the unfolded eigenvalues of $H^\Lambda_\omega$. Then it is seen that the sequence $\{e^\Lambda_\omega(\omega)\}$ is asymptotically uniformly distributed on $[0, |\Lambda|]$ as $L \to \infty$.

**Conjecture 1** Let $\mu$ be the uniform distribution on $[a, b]$. For $\omega \in \Omega$ and $t \in [a, b]$, define a point process

$$\Xi^\omega_{t, \mu}(dx) := \sum_j \delta_{\xi^\omega_j(\omega)-|\Lambda|t}(dx).$$

Then for $P$-a.a. $\omega \in \Omega$, the probability law of $\Xi^\lambda_{t, \mu}$ under $\mu(dt)$ converges weakly to that of the stationary Poisson point process with mean density 1.

In this talk, I showed that if we would be able to prove the following lemma, then one would obtain a weaker version of the conjecture:

**Lemma(also a conjecture)** For any finite intervals $J$ and $E \neq E'$,

$$P(\eta^\omega(C_p; E)(J) \geq 1 \text{ and } \eta^\omega(C_p; E')(J) \geq 1) = o(N^{-d}_L)$$

as $L \to \infty$.

**References**