The energy level statistics for the Anderson tight binding model
- Statement of a conjecture -

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Let us consider the Anderson tight binding model $H_\omega = -\Delta + V_\omega$, where $(\Delta u)(x) = \sum_{|y-x|=1} u(y)$ $(x, y \in \mathbf{Z}^d)$, is the discretized Laplacian, and $V_\omega = \{V_x(\omega)\}_{x\in\mathbf{Z}^d}$: is the random potential consisting of i.i.d. random variables. For $L = 1, 2, \ldots$, let $\Lambda := \Lambda_L = [0, L]^d \cap \mathbf{Z}^d$, and consider $H_\omega^\Lambda := \chi_\Lambda H_\omega \chi_\Lambda$, the restriction of H_ω to the hypercube Λ with the Dirichlet boundary condition. Now let $E_1^\Lambda(\omega) \leq \cdots \leq E_n^\Lambda(\omega)$, $n = |\Lambda|$, be the eigenvalues of H_ω^Λ. At this point, we assume that each random variable $V_x(\omega)$ has absolutely continuous distribution with a bounded density $\rho(v)$. We also assume that its upperbound $\|\rho\|_\infty$ is small. Smallnes of $\|\rho\|_\infty$ is assumed because it implies the exponential decay of the fractional moment of the Green's function $G_\omega^D(z; x, y) = (H_\omega^D - z)^{-1}(x, y)$. (See [1], [2] for the exact formulation.)

We now consider the rescaled spectrum of H_ω^Λ expressed as a point process on $\mathbf{R}$: $\xi^\omega(\Lambda; E)(dx) = \sum_j \delta_{\xi_j^\omega(\Lambda;E)}(dx)$, with $\xi_j^\omega(\Lambda; E) = |\Lambda|(E_j^\Lambda(\omega) - E)$. It was proved in [3] that if the integrated density of states $N(E)$ of H_ω is differentiable at E with $n(E) = dN/dE$, then as $L \to \infty$, the probability law of $\xi^\omega(\Lambda; E)$ converges weakly to that of the stationary Poisson point process with mean density $n(E)$. Then a question arises: Can we compare the individual spectrum $\{E_j^\Lambda(\omega)\}$ of H_ω^Λ with the typical realization of a nice point process on $\mathbf{R}$?

For this purpose, we need to "unfold" the spectrum. Let us assume that $N(E)$ is of C^1 and $n(E) > 0$ everywhere on $(\inf\Sigma, \sup\Sigma)$, where $\Sigma \subset \mathbf{R}$ is the closed set such that $spec(H_\omega) = \Sigma$ a.s.. Now let us call $e_j^\Lambda(\omega) := |\Lambda| \cdot N(E_j^\Lambda(\omega)) \in (0, |\Lambda|)$ the unfolded eigenvalues of H_ω^Λ. Then it is seen that the sequence $\{e_j^\Lambda(\omega)\}$ is asymptotically unformly distributed on $[0, |\Lambda|]$ as $L \to \infty$.

Conjecture1 Let μ be the uniform distribution on $[a, b]$. For $\omega \in \Omega$ and $t \in [a, b]$, define a point process

$$\Xi_{\omega,t}^\Lambda(dx) := \sum_j \delta_{e_j^\Lambda(\omega) - |\Lambda| t}(dx) \ .$$

Then for P-a.a. $\omega \in \Omega$, the probability law of $\Xi_{\omega,t}^\Lambda$ under $\mu(dt)$ converges weakly to that of the stationary Poisoon point process with mean density 1.

In this talk, I showed that if we would be able to prove the following lemma, then one would obtain a weaker version of the conjecture:

Lemma(also a conjecture) For any finite intervals J and $E \neq E'$,

$$P\left(\eta^\omega(C_p; E)(J) \geq 1 \text{ and } \eta^\omega(C_p; E')(J) \geq 1\right) = o(N_L^{-d})$$

as $L \to \infty$.

References

[1] Aizenman, M., Molchanov, S.A.: Commun. Math. Phys. **157**, 245-278 (1993)

[2] Graf, G.M.: J. Stat. Phys. **75**, 337-346 (1994)

[3] Minami, N.: Commun. Math. Phys. **177**, 709-725 (1996)