Schrödinger operators with random δ magnetic fields

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We consider random magnetic Schrödinger operators on $\mathbf{R}^2$

$$H_\omega = \left(\frac{1}{i}\nabla + \mathbf{a}_\omega\right)^2,$$

where $\mathbf{a}_\omega = (a_{\omega,x}, a_{\omega,y})$ is the magnetic vector potential. We consider two types of random magnetic fields:

$$\begin{aligned} \operatorname{rot} \mathbf{a}_\omega(z) &= B + \sum_{\gamma\in\Gamma} 2\pi\alpha_\gamma(\omega)\delta(z-\gamma), \quad \text{(Anderson type)} \\ \operatorname{rot} \mathbf{a}_\omega(z) &= B + \sum_{\gamma\in\Gamma_\omega} 2\pi\alpha\delta(z-\gamma), \quad \text{(Poisson type)} \end{aligned}$$

where $B \geq 0$, $0 < \alpha < 1$ are constants, $\{\alpha_\gamma(\omega)\}_{\gamma\in\Gamma}$ are independently, identically distributed $[0,1)$-valued random variables, Γ is a lattice, and Γ_ω is the Poisson point process with intensity measure $\rho dxdy$ ($\rho > 0$ is a constant).

We studied some spectral properties of these operators. The main statements are summarized as follows:

1. Consider the Anderson case and assume $B = 0$. If the support of the common distribution measure $\mu = P \circ \alpha_\gamma^{-1}$ contains 0 or 1, then the Lifshitz tail holds near the energy $E = 0$.

2. Consider the Poisson case and assume $B > 0$, $\alpha \notin \mathbf{Q}$. Then, $\sigma(H_\omega) = [B, \infty)$ almost surely.

3. Consider the Anderson case and assume $B > 0$, $\operatorname{supp} \mu \not\ni 0, 1$. Then, the lower Landau levels are isolated, infinitely degenerated eigenvalues.

4. Consider both cases and assume $B > 0$. For each $n = 1, 2, \ldots$, there exists a number B_n independent of ω satisfying the following: if $B > B_n$, then the n-th Landau level $E_n = (2n-1)B$ is an infinitely degenerated eigenvalue of H_ω almost surely.

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