The distribution of eigenfunctions in the Anderson model

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Abstract : In the Anderson model, we consider the random measure composed of its eigenvalues and eigenfunctions and show that, it converges in the natural scaling limit to the Poisson process in the product space of energy and space. In another scaling, the eigenfunctions are repulsive each other. Some related problem are also discussed.

We consider the so-called Anderson model on $l^2(\mathbf{Z}^d)$ defined by

$$(H_\omega \phi)(x) = \sum_{|y-x|=1} \phi(y) + V_\omega \phi(x), \ quad \phi \in l^2(\mathbf{Z}^d)$$

where $\{V_\omega(x)\}_{x \in \mathbf{Z}^d}$ are independent, identically distributed random variables on some probability space $(\Omega, \mathcal{F}, \mathbf{P})$ with the distribution of $V_\omega(0)$ having the bounded density ρ. It is known that we can find an open interval I in the spectrum of H_ω with Anderson localization holds (dense pure point spectrum with exponntially decaying eigenfunctions). We first introduce the random measure defined by

$$\xi(J \times B) := \mathrm{Tr}\ (1_B(x) 1_J(H_\omega) 1_B(x)), \quad J \subset \mathbf{R}, B \subset \mathbf{R}^d.$$

To study the local distribution of the eigenvalues and corresponding eigenfunctions, we take the reference energy $E_0 \in I$ and consider the "natural scaling" ξ_L given by

$$\int \int f(E, x) d\xi_L := \int \int f(L^d(E - E_0), x/L) d\xi, \quad f \in C_c(\mathbf{R}^{d+1}).$$

Let ν be the density of states measure

$$\nu(B) := \mathbf{E}[\langle 0 | 1_B(H_\omega) | 0 \rangle], \quad B \in \mathcal{B}(\mathbf{R}).$$

Theorem 1 (with R. Killip) *Suppose $E_0 \in I$ is the Lebesgue point of the density of states measure ν. Then ξ_L converges in distribution to the Poisson process on $\mathbf{R}^{d+1}$ with intensity measure $\frac{d\nu}{dE}(E_0) dE \times dx$.*

It is well-known that the point process on $\mathbf{R}$ composed of eigenvalues of H in finite-volume approximation converges to the Poisson process on $\mathbf{R}$ [2]. Theorem 1 can be regarded as an extension of that. For other models (e.g., for the Schrödinger operators on $l^2(\mathbf{Z}^d)$ with off-diagonal disorder or with random fluxes, or for the Schrödinger operators in the continuum), we can show ξ_L has convergent subsequences with infinitely divisible limting point process. It is also possible to consider the point process composed by the eigenvalues and corresponding localization centers, in which we have similar results. In another scaling, something different happens [3] : if the energies E, E' satisfy $|E - E'| \simeq L^{-2d}$, then the corresponding localization centers $x(E), x(E')$ must satify $|x(E) - x(E')| > L$. In all works, Minami's estimate [2] is the essential ingredient of the proofs.

References

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