Games of singular control and stopping driven by spectrally one-sided Lévy processes

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We study a zero-sum game where the evolution of a spectrally one-sided Lévy process is modified by a singular controller and is terminated by the stopper. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space hosting a spectrally one-sided Lévy process $X = \{X_t; t \geq 0\}$. Let $\mathbb{P}_x$ be the conditional probability under which $X_0 = x$ (also let $\mathbb{P} \equiv \mathbb{P}_0$), and let $\mathbb{F} := \{\mathcal{F}_t : t \geq 0\}$ be the filtration generated by X.

The games analyzed in this paper consist of two players. The controller chooses a process $\xi \in \Xi$, which denotes the set of *nondecreasing* and *right-continuous* $\mathbb{F}$-adapted processes with $\xi_{0-} := 0$, while the stopper chooses the time $\tau \in \Upsilon$ among the set of $\mathbb{F}$-stopping times Υ. The controller minimizes and the stopper maximizes the common performance criterion:

$$J(x; \xi, \tau) := \mathbb{E}_x \left[\int_0^\tau e^{-qt} h(U_t^\xi) \mathrm{d}t + \int_{[0,\tau]} e^{-qt} \mathrm{d}\xi_t + e^{-q\tau} g(U_\tau^\xi) 1_{\{\tau < \infty\}} \right],$$

where U^ξ is a right-continuous controlled process defined by

$$U_t^\xi := X_t + \xi_t, \quad t \geq 0.$$

The problem is to show the existence of a *saddle point*, or equivalently a *Nash equilibrium* $(\xi^*, \tau^*) \in \Xi \times \Upsilon$, such that

$$J(x; \xi^*, \tau) \leq J(x; \xi^*, \tau^*) \leq J(x; \xi, \tau^*), \tag{0. 1}$$

for any $\xi \in \Xi$ and any stopping time $\tau \in \Upsilon$; in this case we call $J(x; \xi^*, \tau^*)$ the value function of the game.

In this paper, we consider for X a general spectrally negative or positive Lévy process and show under a suitable condition that a saddle point is given by $(\xi^a, \tau_{a,b})$ for some $a < b$, where we define

$$\begin{aligned} \xi^a_t &:= \sup_{0 \leq t' \leq t} (a - X_{t'}) \vee 0, \quad t \geq 0, \\ \tau_{a,b} &:= \inf\{t \geq 0 : U^{\xi^a}_t > b\}. \end{aligned}$$

The saddle point and the corresponding value function is written in terms of the scale function. Numerical examples under phase-type Lévy processes are also given.

References

[1] Hernández-Hernández, D. and Yamazaki, K., *Games of singular control and stopping driven by spectrally one-sided Lévy processes*, arXiv:1308.3141, 2013.