# 複素力学系とその周辺分野の研究

Research on Complex Dynamics and Related Fields

## (宇敷重廣教授還暦記念集会)

### 研究集会

京都大学数理解析研究所の共同研究事業で下記のように研究集会を催しますので、ご案内申し上げます。なお本研究集会は長く日本の力学系・複素力学系グループを牽引・指導してこられました宇敷重廣教授(京大・人間・環境)のご還暦を記念して開催されます。

研究代表者 角 大輝 (大阪大学 大学院理学研究科)

記

日時:2010年 12月 6日(月)13:30~

12月 10日(金)12:30

場所: 京都大学理学部3号館110講義室(数理解析研究所ではありません)

京都市左京区北白川追分町(市バス百万遍または京大農学部前下車)

#### プログラムと講演要旨

12月6日(月)

13:30~14:30 田中 剛平 (Gohei Tanaka, 東大・生産技術研究所)

Nonlinear information processing with complex-valued neural networks

要旨: Complex-valued neural networks, described by high-dimensional complex dynamical systems, have been used for information processing in computational intelligence. We first review the history and the framework of complex-valued neural networks. Then we present modifications of the complex activation function based on nonlinearity and its applications.

14:45~15:45 神 貞介 (Teisuke Jin, 京大)

Dynamics of the Hénon map: Nevanlinna Theory

要旨: Some new applications of Nevanlinna theory to the dynamics of the Hénon map.

16:00~17:00 内村 桂輔 (Keisuke Uchimura, 東海大・理)

Dynamics of Generalized Chebyshev Maps

要旨: 以前の結果に付け加えて、Critical measure の slicing について.

12月7日(火)

10:15~10:45 片方 江 (Koh Katagata, 一関高専)

Limits of entire functions with respect to some metric

要旨: 整関数全体の集合の部分集合に特異値の軌道を考慮した距離を導入し, その距離に関する極限関数の力学系を考察します.

11:00~11:30 諸澤 俊介 (Shunsuke Morosawa, 高知大・理)

Some questions of Schröder functions

13:30~14:30 木坂 正史 (Masashi Kisaka, 京大・人間・環境)

Smoothness of hairs for some transcendental entire functions

要旨: 指数関数の Julia 集合には hair と呼ばれる構造があり,それは  $C^\infty$  級の曲線であることが知られている.この講演では指数関数を含むあるクラスの超越整関数に対し,Julia 集合に hair が存在し,それが  $C^\infty$  級の曲線であることを示す.

14:45~15:45 稲生 啓行 (Hiroyuki Inou, 京大・理)

Holomorphic index and parabolic renormalization

要旨: 放物型くりこみの定義域と値域に関する (簡単な) 補題について話します.

16:00~17:00 篠原 知子 (Tomoko Shinohara, 都立産業技術高専)

Locally maximal invariant set at a fixed indeterminate point

要旨: 複素 3 次元有理写像の固定的不定点に存在する局所最大不変集合を blow up を用いて構成する.特に,不定点集合が一次元の場合を 議論する.

12月8日(水)

9:30~10:30 上野 康平 (Kohei Ueno, 鳥羽商船高専)

Symmetries of Julia sets of polynomial skew products on  $\mathbb{C}^2$ 

要旨: A Julia set of a polynomial skew product can have symmetries, that is, it can be invariant under some polynomial automorphisms which are conjugate to rotational products. We investigate the structure of the group of symmetries and give a necessary and sufficient condition for the group of symmetries to be infinite.

10:45~11:45 小木曽 啓示 (Keiji Oguiso, 大阪大・理)

Entropy of automorphisms of compact hyperKaeler manifolds

#### 13:30~14:30 角 大輝 (Hiroki Sumi, 大阪大・理)

Cooperation principle and density of stable systems in random complex dynamics

要旨: We investigate the i.i.d. random dynamics of rational maps and the dynamics of semigroups of rational maps on the Riemann sphere  $\hat{\mathbb{C}}$ . We show that regarding random complex dynamics of polynomials, generically, the chaos of the averaged system disappears, due to the cooperation of the generators. We investigate the iteration and spectral properties of transition operators acting on the space of (Hölder) continuous functions on  $\hat{\mathbb{C}}$ . We also investigate the stability and bifurcation of random complex dynamics. We show that the set of stable systems is open and dense in the space of random dynamics of polynomials. Moreover, we prove that for a stable system, there exist only finitely many minimal sets, and each minimal set is attracting. These results correspond to solving a kind of analogy of the famous conjecture "regarding the usual iteration, hyperbolic maps are dense in the space of polynomial maps". Moreover, we prove that the orbit of a Hölder continuous function on  $\hat{\mathbb{C}}$  under the transition operator tends exponentially fast to the finite-dimensional space U of finite linear combinations of unitary eigenvectors of the transition operator. Thus the spectrum of the transition operator acting on the space of Hölder continuous functions has a gap between the set of unitary eigenvalues and the rest. Combining this with the perturbation theory for linear operators, we obtain that for a stable system constructed by a finite family of rational maps, the projection to the space U depends real-analytically on the probability parameters. By taking a partial derivative of the function of probability of tending to a minimal set with respect to a probability parameter, we obtain a complex analogue of the Takagi function. For the references, see H. Sumi, Cooperation principle, stability and bifurcation in random complex dynamics, preprint 2010, http://arxiv.org/abs/1008.3995, and H. Sumi, Random complex dynamics and semigroups of holomorphic maps, to appear in Proc. London Math. Soc., http://arxiv.org/abs/0812.4483.

14:45~15:45 Eric Bedford (Indiana Univ.)

Linear fractional recurrences as birational maps of 3-space: Periodicities and pseudo-automorphisms of positive entropy

Abstract: We consider the family of 3-step linear fractional recurrences

$$z_{n+3} = \frac{\alpha_0 + \alpha_1 z_n + \alpha_2 z_{n+1} + \alpha_3 z_{n+2}}{\beta_0 + \beta_1 z_n + \beta_2 z_{n+1} + \beta_3 z_{n+2}}, \quad \alpha_i, \beta_i \in \mathbf{C}$$

which induce a family of birational maps  $f_{\alpha\beta}$  of 3-space. We say that a birational map is a pseudo-automorphsm if neither f nor  $f^{-1}$  has an exceptional hypersurface. By examining the orbits of exceptional hypersurfaces, we see that if  $f_{\alpha\beta}$  is a pseudo-automorphism, then f is equivalent to the case  $\alpha_2 \neq 0$ ,  $\beta_1 = \alpha_3 = 1$ ,  $\beta_2 = \beta_3 = 0$ . We show that the only possible periods for recurrences of this form are 8 and 12.

We show, too, that there is a family of maps of this form which gives pseudo-automorphisms of positive entropy, and we examine the structure of these maps. This is joint work with Kyounghee Kim.

16:00~17:00 宇敷 重廣 (Shigehiro Ushiki, 京大・人間・環境)

Exploration of complex Hénon dynamics

要旨: Complicated entanglements of invariant manifolds are visualized by means of interactive computer graphics. New version of Complex-Explorer, combined with StereoViewer, provides a tool for the investigation of the parameter space and the dynamic space of complex He'non family. (N.B. ComplexExplorer and StereoViewer are graphics software.)

18:00~ 宇敷先生ご還暦祝賀会ならびに懇親会 (京都大学百周年時計台記念館 国際交流ホール III (京都大学時計台下))

12月9日(木)

10:30~11:10 今田 光彦 (Mitsuhiko Imada, 東工大・理工)

Periodic points on the boundaries of rotation domains

要旨: 有理関数のジーゲル円板の境界が周期点を含み得るのはどのような場合であるかについて考える。具体的には、ジーゲル円板の境界の近傍で単射であるならば、その境界は放物・反発周期点は含まないことを、いくつかの概念や既知の結果を組み合わせることによって証明する。

11:25~12:25 中根 静男 (Shizuo Nakane, 東京工芸大・工)

Postcritical sets and saddle basic sets for Axiom A polynomial skew products on  $\mathbb{C}^2$ 

要旨:  $\mathbb{C}^2$  の Axiom A polynomial skew products の力学系について、特に危点集合の集積点集合の性質を saddle basic sets とその安定集合を用いて記述し、Demarco-Hruska の結果との関係について言及する。

14:00~15:00 宍倉 光広 (Mitsuhiro Shishikura, 京大・理)

A case study of an application of Thurston's theorem

15:15~16:15 上田 哲生 (Tetsuo Ueda, 京大・理)

Critically finite maps on projective spaces

要旨: 複素射影空間上の strictly critically finite map について反発周期点が稠密であることを示す.

16:30~17:30 足助 太郎 (Taro Asuke, 東大・数理)

On Fatou-Julia decompositions

要旨: According to Sullivan's dictionary, Fatou sets and domains of discontinuity (of Kleinian groups) are in a close relationship. Fatou sets can be defined also for non singular holomorphic foliations and they enjoy properties similar to those of Fatou sets (in a usual sense) or domains of discontinuity. In this talk, we will explain that they can be considered as the same object, and that Fatou sets can be defined for other kinds of dynamical systems, e.g., singular holomorphic foliations or 'pseudosemigroups'.

#### 12月10日(金)

10:15~11:15 荒井 迅 (Zin Arai, 北大・創成研究機構)

Monodromy and bifurcations of the Hénon map

要旨: In this talk, we study the structure of the parameter space of the Hénon map. We show that the monodromy action is determined by the arrangement of bifurcation curves of periodic points, and using this fact, we prove that odd iterations of the shift map can not appear in the image of the monodromy homomorphism. These results are immediately translated into their "real" counterparts via a theorem relating the monodromy of the complex Hénon map and the pruning front of the real Hénon map.

11:30~12:30 John Hubbard (Cornell Univ.)

Topological models for complex Henon mappings: the pinched ball model (joint work with Remus Radu and Raluca Tanase)

なお,プログラムには多少,変更の可能性があります.最新のプログラムと講演要旨は次のホームページをご覧ください:

http://www.math.sci.osaka-u.ac.jp/~sumi/cpxdyn10meeting.html

また、宇敷先生ご還暦祝賀会ならびに懇親会に参加を希望されます方は、11月24日 (水) までに角大輝 sumi(@)math.sci.osaka-u.ac.jp まで氏名、所属、電子メールアドレスなどの連絡先をご連絡ください。

Date: December 4, 2010.