Erratum to 'Semi-hyperbolic fibered rational maps and rational semigroups' (Ergodic Theory and Dynamical Systems 26 (2006), 893-922)

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Abstract

We give a correction to the assumption of Theorem 1.12 and Theorem 2.6 in the paper [H. Sumi. Semi-hyperbolic fibered rational maps and rational semigroups. *Ergod. Th. & Dynam. Sys.* **26** (2006), 893-922].

1 Correction to Theorem 1.12 in [S1]

We use the same notation as that in [S1].

Let $(\pi, Y = X \times \overline{\mathbb{C}}, X)$ be a trivial $\overline{\mathbb{C}}$-bundle. Let $f : Y \to Y$ be a fibered polynomial map over $g : X \to X$ (see [S1, Definition 1.11]). For each $x \in X$, we set $A_x(f) := \{y \in Y_x \mid \pi_{\overline{\mathbb{C}}}(f_x^n(y)) \to \infty\}$ and $K_x(f) := Y_x \setminus A_x(f)$. Moreover, we denote by $\text{int}K_x(f)$ the set of all interior points of $K_x(f)$ with respect to the topology in Y_x.

Theorem 1.12 in [S1] should be replaced by the following form.

Theorem 1.1. *Let $(\pi, Y = X \times \overline{\mathbb{C}},\ X)$ be a trivial $\overline{\mathbb{C}}$-bundle. Let $f : Y \to Y$ be a semi-hyperbolic fibered polynomial map over $g : X \to X$ such that $d(x) \geq 2$, for any $x \in X$. Assume that either:*

(1) $J_x(f)$ is connected for each $x \in X$; or

(2) the map $x \mapsto f_x$ from X to the space of polynomials is locally constant (here, we identify f_x with a polynomial on $\overline{\mathbb{C}}$),
$\inf\{d(a, b) \mid a \in \pi_{\overline{\mathbb{C}}}(J_x(f)),\ b \in \pi_{\overline{\mathbb{C}}}(A_x(f) \cap C(f)),\ x \in X\} > 0,$
and for each $z \in Y$ with $z \in \text{int}K_{\pi(z)}(f)$, there exists no sequence $\{z_j\}_{j=1}^{\infty}$ in Y with $z_j \in A_{\pi(z_j)}(f)$ $(\forall j)$ such that $z_j \to z$ as $j \to \infty$.

Then, there exists a positive constant c such that, for any $x \in X$, the basin of infinity $A_x(f)$ is a c-John domain.

Remark 1. We have added an extra assumption in condition (2).

Remark 2. In the proof of Theorem 1.1, we need the following Claim $(*)$, which does not hold in general when we have condition (2) in [S1, the original Theorem 1.12] instead of the condition (2) of Theorem 1.1 :

Claim $(*)$: Under the assumption of Theorem 1.1, for each $(x, y) \in X \times \mathbb{C}$ let $G_x((x, y)) := \lim_{n\to\infty}(1/d_n(x)) \log^+ |\pi_{\overline{\mathbb{C}}}(f_x^n((x, y)))|$ (see [S1, p. 909]). Moreover, for each $x \in X$ and $z \in Y_x$, let $\delta_x(z) := \inf_{w\in J_x(f)}\{|\pi_{\overline{\mathbb{C}}}(z) - \pi_{\overline{\mathbb{C}}}(w)|\}$. Let $a_1 > 0$ be a number. Then, there exists a positive number a_2 such that if $(x, y) \in X \times \mathbb{C}$, $(x, y) \in A_x(f)$, and $\delta_x((x, y)) > a_1$ then $G_x((x, y)) > a_2$.

proof of Claim $(*)$: Under the assumption of Theorem 1.1, suppose that there exists a sequence $\{z_j\}_{j=1}^{\infty}$ in Y such that for each $j, z_j \in A_{\pi(z_j)}(f)$ and $\delta_{\pi(z_j)}(z_j) > a_1$, and such that $G_{\pi(z_j)}(z_j) \to 0$ as $j \to \infty$. We will deduce a contradiction. We may assume that there exists a point $z \in X \times \mathbb{C}$ such that $z_j \to z$. Since $(x, y) \mapsto G_x((x, y))$ is continuous in $X \times \mathbb{C}$ ([S1, p. 910]) and G_x in $A_x(f) \setminus \{\infty\}$ is harmonic (under the canonical identification $Y_x \cong \overline{\mathbb{C}}$), it follows that $G_{\pi(z)}$ is identically zero in a neighborhood of z in $Y_{\pi(z)}$. Hence $z \in \text{int}K_{\pi(z)}(f)$. From the assumption of Theorem 1.1, it follows that for each $x \in X$, $J_x(f)$ is connected. Since $z \in \text{int}K_{\pi(z)}(f) \subset F_{\pi(z)}(f)$, [S2, Theorem 2.14] implies that there exists a neighborhood U of z in Y_z such that

$$\text{diam} f_{\pi(z)}^n(U) \to 0 \text{ as } n \to \infty \tag{1}$$

and

$$\overline{\cup_{n\in\mathbb{N}} f_{\pi(z)}^n(U)} \subset \tilde{F}(f). \tag{2}$$

By (1), we obtain

$$d(\text{CV}(f_{\pi(z)}^n), f_{\pi(z)}^n(z)) \to 0 \text{ as } n \to \infty, \tag{3}$$

where $\text{CV}(\cdot)$ denotes the set of all finite critical values. Combining (2) and (3), we get that there exists an $n \in \mathbb{N}$ such that the point $f_{\pi(z)}^n(z)$ and a point $v \in \text{CV}(f_{\pi(z)}^n)$ belong to a component V of $F_{g^n(\pi(z))}(f)$. Since $x \mapsto J_x(f)$ is continuous (see [S2, Theorem 2.14]), it follows that there exists an $m \in \mathbb{N}$ such that the point $f_{\pi(z_m)}^n(z_m)$ and a point $v_m \in \text{CV}(f_{\pi(z_m)}^n)$ belong to a component V_m of $F_{g^n(\pi(z_m))}(f)$. However, since $f_{\pi(z_m)}^n(z_m) \in A_{g^n(\pi(z_m))}(f)$ and $J_x(f)$ is connected for each $x \in X$, it causes a contradiction. Thus we have proved Claim $(*)$.

Example 1.2. Let $c \in \mathbb{C}, |c| > 6$. Let $h_1(z) := z^2 + c$ and $h_2(z) := z^2 - c$. Let $f : \Sigma_2 \times \overline{\mathbb{C}} \to \Sigma_2 \times \overline{\mathbb{C}}$ be the fibered rational map associated with the generator system $\{h_1, h_2\}$ (see [S1, Definition 1.4]). Then, f is a hyperbolic fibered polynomial map over the shift map $\sigma : \Sigma_2 \to \Sigma_2$ and f satisfies condition (2) in Theorem 1.1. In fact, for each $x \in \Sigma_2$, $\text{int}K_x(f) = \emptyset$.

2 Correction to Theorem 2.6 in [S1]

In [S1, Theorem 2.6], the sequence (n_j) of $\mathbb{N}$ should be strictly increasing.

References

[S1] H. Sumi, *Semi-hyperbolic fibered rational maps and rational semigroups*, Ergod. Th. & Dynam. Sys. **26** (2006), 893-922

[S2] H. Sumi, *Dynamics of sub-hyperbolic and semi-hyperbolic rational semigroups and skew products*, Ergod. Th. & Dynam. Sys. **21** (2001), 563-603.