Volume **120**, Number 1 (1994), in the article "Lower bounds for the spectra of the Schrödinger operators with magnetic fields," by Naomasa Ueki, pages 344–379.

The proof of Lemma 5.1 is incorrect, since (5.9) should be corrected as

$$\|e^{-L^n(b)}\|_{\mathrm{End}(L^2(M))} = \sup_{\substack{\varphi\in C_0(M)\\ \|\varphi\|_{L^2(M)}=1}} \|e^{-L^n(b)}\varphi\|_{L^2(M)}. \tag{1}$$

This mistake affects all the mechanism of the proof of Section 5.

However, if the manifold M is compact, then the results of Section 5 hold true.

Theorem 5.1'. *If the manifold M is compact, then*

$$\varliminf_{\xi\to\infty} \frac{\inf \mathrm{spec}\ L(\xi b)}{\xi} \geq \frac{1}{4} \inf_{x\in M} \|db(x)\|_1. \tag{2}$$

Proof. We take $\delta > 0$ arbitrarily. Since M is compact, we can take a finite open covering $\{D_\iota\}_{\iota=1}^N$ of M and a family of C^2 functions $\{h_\iota\}_{\iota=1}^N$ satisfying the following:

(i) for each ι, the diameter of D_ι is less than δ and, on $\overline{D_\iota}$, we can take subbundles $U^\iota, V^\iota, W^\iota$ and an almost complex structure J^ι so that (2.2) holds and $\mathcal{B}^\iota(x_\iota) = \|db(x_\iota)\|_1/4$ for some $x_\iota \in D_\iota$, (3) where $\mathcal{B}^\iota$ is the corresponding $\mathcal{B}$ defined in (2.4),

(ii) $\mathrm{supp} h_\iota \subset D_\iota$ and $\sum_\iota (h_\iota)^2 \equiv 1$.

Then we obtain

$$L(\xi b) = \sum_\iota h_\iota L(\xi b) h_\iota - \frac{1}{2}\sum_\iota \|dh_\iota\|^2 \tag{4}$$

(cf. B.Simon, Ann.Inst.Henri Poincaré, Physique Théorique, **38** (1983), 295–307). Noting

$$\sup_{x\in M} \sum_\iota \|dh_\iota(x)\|^2 < \infty, \tag{5}$$

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we estimate as

$$\text{(6)} \qquad \inf \text{spec } L(\xi b) \geq \inf_{\iota} \inf \text{spec } L(\xi b)_{D_\iota} - \frac{1}{2} \sup_{x \in M} \sum_{\iota} \|dh_\iota(x)\|^2,$$

where $L(\xi b)_{D_\iota}$ is the corresponding $L(\xi b)$ on $\overline{D_\iota}$. By applying Theorem 2.1, we have

$$\text{(7)} \qquad \inf \text{ spec } L(\xi b)_{D_\iota} \geq - \sup_{y \in D_\iota} \sum_{j=1}^{3} \mathcal{R}_j^\iota(y) + \xi \inf_{y \in D_\iota} \mathcal{B}^\iota(y),$$

where $\mathcal{R}_j^\iota$ are the corresponding $\mathcal{R}_j$ defined in (2.3). Therefore we obtain

$$\text{(8)} \qquad \varliminf_{\xi \nearrow \infty} \frac{\inf \text{spec } L(\xi b)}{\xi} \geq \inf_{\iota} \inf_{y \in D_\iota} \mathcal{B}^\iota(y).$$

Since δ was arbitrary, we obtain (2). ∎

The derivation of Corollaries 1 and 2 from Theorem 5.1 is correct. This proof is valid more generally under the situation that, for any $\delta > 0$, we can take a locally finite open covering $\{D_\iota\}$ and a family of C^2 functions $\{h_\iota\}$ satisfying (3) and (5), and the assumptions (A.2), (A.3) hold.